UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 110A
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December 9, 1991

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FINAL

SHOW YOUR WORK AS CLEARLY AS POSSIBLE

The exam is closed book and notes, but you may use any information you have brought with you on one side of a $8\frac{1}{2}'' \times 11''$ sheet of paper. Do not spend too much time on one problem. Be sure to give answers in terms of given quantities.

#	Pts
1	15
2	15
3	11
4	12
5	14
6	14
7	15
	96

Good Work!

COURSE A+

Colley

1. A sphere of radius a contains charge of constant density:

$$\rho = \rho_0 \qquad r < a$$
$$\rho = 0 \qquad r > a$$

a) Find ϕ everywhere.

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b) Calculate the electrical energy stored, using $U_e = \frac{1}{2}\int \rho\phi d\tau$.

c) Calculate the electrical energy stored, using $U_e = \frac{1}{2}\int \epsilon_0 E^2 d\tau$.

a) $r > a$ $\boxed{\phi = \frac{Q_{in}}{4\pi\epsilon_0 r} = \frac{\rho_0 a^3}{3\epsilon_0 r}}$

5

$Q_{in} = \frac{4}{3}\pi a^3 \rho_0$

$r < a$

$\vec{E} = \frac{Q_{in}}{4\pi\epsilon_0 r^2}$ $\quad Q_{in} = \frac{4}{3}\pi r^3 \rho_0$ $\quad \vec{E} = \frac{\rho_0 r}{3\epsilon_0}$ $\quad \phi(a) = \frac{\rho_0 a^2}{3\epsilon_0}$

$\phi(r) = \int_r^a \vec{E}\cdot d\vec{r} + \phi(a) = \frac{\rho_0}{3\epsilon_0}\int_r^a r\,dr = \frac{\rho_0}{6\epsilon_0} r^2\Big|_r^a = \frac{\rho_0}{6\epsilon_0}(a^2 - r^2)$

$\Rightarrow \boxed{\phi(r) = \frac{\rho_0}{6\epsilon_0}(a^2 - r^2) + \frac{\rho_0 a^2}{3\epsilon_0}}$

$\frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \frac{2}{15}$

5 b) $U_e = \frac{1}{2}\int_V \rho\phi\, d\tau = \frac{1}{2}\rho_0 \int_0^{2\pi}\int_{-1}^{1}\int_0^a \left(\frac{\rho_0}{6\epsilon_0}(a^2 - r^2) + \frac{\rho_0 a^2}{3\epsilon_0}\right) r^2 dr\, d\cos\theta\, d\phi$

$= \frac{4\pi\rho_0^2}{2\cdot 6\epsilon_0}\int_0^a (a^2 - r^2) r^2 dr + \frac{4\pi\rho_0^2 a^2}{6\epsilon_0}\int_0^a r^2 dr = \frac{4\pi\rho_0^2}{12\epsilon_0}\left[\frac{a^2 r^3}{3}\Big|_0^a - \frac{r^5}{5}\Big|_0^a\right] + \frac{4\pi\rho_0^2 a^2 a^3}{6\epsilon_0 3}$

$\frac{2a^3}{15}$

$= \frac{4\pi\rho_0^2}{12\epsilon_0}\left[\frac{a^5}{3} - \frac{a^5}{5}\right] + \frac{4\pi\rho_0^2 a^5}{6\epsilon_0 3} = \frac{\rho_0^2 a^5 \pi}{\epsilon_0}\left(\frac{8^2}{12\cdot 15} + \frac{4}{18}\right) = \boxed{\frac{12}{45}\frac{\rho_0^2 a^5 \pi}{\epsilon_0}}$

$\frac{12}{45}$

Work Space

5

c) $U_e = \frac{1}{2}\epsilon_0 \int_V E^2 \, d\tau'$ $\qquad r>a \quad \vec{E} = \frac{\rho_0 a^3}{3\epsilon_0 r^2}$

$\qquad r<a \quad \vec{E} = \frac{\rho_0 r}{3\epsilon_0}$

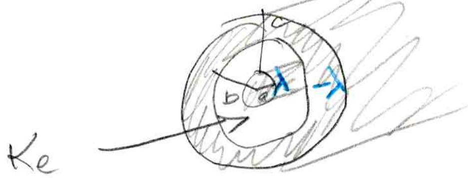
$$= \frac{1}{2}\epsilon_0 \int_0^{2\pi}\int_{-1}^{1}\int_0^{a} \left(\frac{\rho_0 r}{3\epsilon_0}\right)^2 r^2 \, dr \, d\cos\theta \, d\phi + \frac{1}{2}\epsilon_0 \int_0^{2\pi}\int_{-1}^{1}\int_a^{\infty} \left(\frac{\rho_0 a^3}{3\epsilon_0 r^2}\right)^2 r^2 \, dr \, d\cos\theta \, d\phi$$

$$= \frac{2\pi \cancel{\epsilon_0} \rho_0^2}{9\epsilon_0^{\cancel{2}}} \int_0^a r^4 \, dr + \frac{2\pi \cancel{\epsilon_0} \rho_0^2 a^6}{9\epsilon_0^{\cancel{2}}} \int_a^{\infty} \frac{1}{r^2} \, dr \qquad r^{-2} \quad -\frac{1}{r}\Big|_a^{\infty} = \frac{1}{a}$$

$$\frac{r^5}{5}\Big|_a^a$$

$$= \frac{2\pi\rho_0^2 a^5}{45\epsilon_0} + \frac{5\cdot 2\pi\rho_0^2 a^5}{5\cdot 9\epsilon_0} = \boxed{\frac{12}{45}\frac{\rho_0^2 a^5 \pi}{\epsilon_0}}$$

Kelley



2. A coaxial cable consists of two conductors, one with outer radius a and the second with inner radius $b > a$, outer radius c. The gap between the conductors ($a < \rho < b$) is filled with a dielectric with dielectric constant κ_e. The inner conductor carries linear charge density λ, the outer carries density -λ.

   a) What is $\vec{E}$ between the conductors?

   b) What is the capacitance per unit length?

   c) What is the volume bound charge density in the dielectric (ρ_b)?

   d) What is the surface bound charge density, σ_b, on the outer surface of the dielectric?

   e) What is the energy stored per unit length?

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3 a) $\vec{D} = \frac{\lambda}{2\pi\rho}\hat{\rho} \Rightarrow \vec{E} = \frac{\vec{D}}{K_e\epsilon_0} = \boxed{\frac{\lambda}{2\pi\epsilon_0\rho K_e}\hat{\rho}}$

b) $C = \frac{Q}{\Delta\phi}$ $\quad \Delta\phi = +\int_a^b E_\rho d\rho = +\frac{\lambda}{2\pi\epsilon_0 K_e}\int_a^b \frac{d\rho}{\rho} = +\frac{\lambda}{2\pi\epsilon_0 K_e}\ln\frac{b}{a}$

3

$Q = \lambda L$

$\frac{C}{L} = \frac{\cancel{\lambda} L\, 2\pi\epsilon_0 K_e}{\cancel{\lambda}\ln(b/a)} = \boxed{\frac{2\pi\epsilon_0 \cancel{L} K_e}{\ln(b/a)}}$

3 c) $\vec{P} = (K_e - 1)\epsilon_0\vec{E} = \frac{(K_e-1)}{K_e}\frac{\lambda}{2\pi\rho}\hat{\rho}$

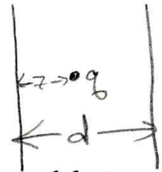
$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho P_\rho) = \boxed{0}$

3 d) $\sigma_b = P_{\rho=b}\cdot\hat{n} = \boxed{\frac{(K_e-1)}{K_e}\frac{\lambda}{2\pi b}}$

inner sum 0?

3 e) $\frac{U_e}{L} = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}\, d\tau$ $\quad \vec{D} = \frac{\lambda}{2\pi\rho}\hat{\rho}$ $\quad \vec{E} = \frac{\lambda}{2\pi\epsilon_0\rho K_e}\hat{\rho}$

$= \frac{\lambda^2}{8\pi^2\epsilon_0 K_e}\int_0^L\int_0^{2\pi}\int_a^b \frac{1}{\rho^2}\rho\, d\rho\, d\phi\, dz = \frac{L\lambda^2}{4\pi\epsilon_0 K_e}\ln\rho\Big|_a^b = \boxed{\frac{\cancel{L}\lambda^2\ln(b/a)}{4\pi\epsilon_0 K_e}}$



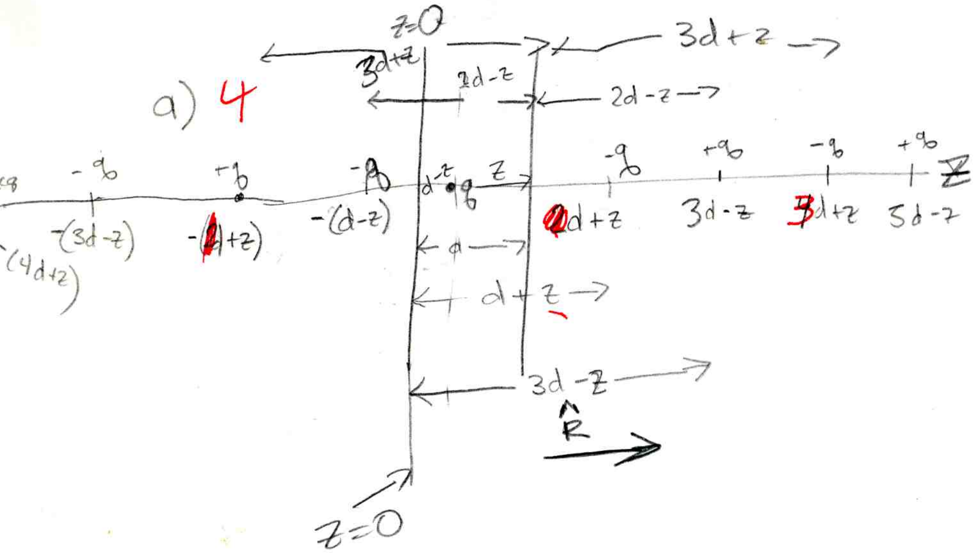
$$F = \frac{qq'}{4\pi\epsilon_0 R^2}\hat{R}$$

3. A point charge q is located between two parallel, grounded conductivity planes which are separated by distance d. The charge is a distance $z < d$ from one plane.

11(a) Find the location of the infinite number of image charges.

b) Express the force on q by an infinite series.

a) 4



for $z > 0$

$(-1)^{l+1} q$ is located at

$$\boxed{ld + (-1)^l z}$$

summed from $l=2$ to $l=\infty$

for $z < 0$

$(-1)^l q$ is located at

$$\boxed{-(ld + (-1)^l z)}$$

summed from $l=1$ to $l=\infty$

7 b)
$$F = \frac{q^2}{4\pi\epsilon_0}\left[\sum_{l=2}^{\infty}\frac{(-1)^{l+1}}{\left[ld + (-1)^l z - (d-z)\right]^2}(-\hat{R}) + \sum_{l=1}^{\infty}\frac{(-1)^l}{\left[ld + (-1)^l z + (d-z)\right]^2}\hat{R}\right]$$

$$= \frac{q^2}{4\pi\epsilon_0}\left[\sum_{l=2}^{\infty}\frac{(-1)^l}{\left[(l-1)d + \{(-1)^l + 1\}z\right]^2}\hat{R} + \sum_{l=1}^{\infty}\frac{(-1)^l}{\left[(l+1)d + \{(-1)^l - 1\}z\right]^2}\hat{R}\right]$$

4. A circular ring of radius a lies in the x y plane with origin at the center, with constant linear charge density $q/2\pi a$.

   a) Find the potential $\phi(z)$ on the z-axis.

   b) Find the potential $\phi(r,\theta)$ $(r > a)$ expressed as a series in $P_\ell(\cos\theta)$. Give the coefficients for $\ell \leq 2$.

   c) If we place a negative charge at the origin, is it in stable equilibrium?

   d) Calculate the quadrupole moment Q^a of this charge distribution. ($Q^a = Q^a_{zz}$ for an axially symmetric distribution.)

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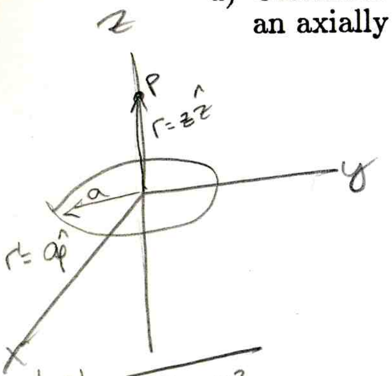


$|R| = \sqrt{a^2+z^2}$

4

a) $$\phi(\vec{z}) = \frac{1}{4\pi\epsilon_0}\int_L \frac{\lambda(\vec{r})\, d\vec{s}}{R}$$

$$= \frac{q}{2\pi a}\int_0^{2\pi} \frac{a\, d\phi}{\sqrt{a^2+z^2}} = \boxed{\frac{q}{4\pi\epsilon_0\sqrt{a^2+z^2}}}$$

b) $$\phi(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{(\ell+1)}}\right)P_\ell(\cos\theta)$$

$$\phi(r=z,\theta=0) = \frac{q}{\sqrt{a^2+z^2}}$$ which can be expanded:

$z > a$ $$\frac{q}{\sqrt{a^2+z^2}} = \frac{q}{z}\frac{1}{\sqrt{\frac{a^2}{z^2}+1}} = \frac{q}{z}\left(1+\frac{a^2}{z^2}\right)^{-1/2} = \frac{q}{z}\left(1 - \frac{1}{2}\frac{a^2}{z^2} + \frac{3}{8}\frac{a^4}{z^4} - \frac{5}{16}\frac{a^6}{z^6} + \dots\right)$$

$$\phi(\theta=0) = \sum_{\ell=0}^{\infty}\left(A_\ell z^\ell + \frac{B_\ell}{z^{\ell+1}}\right)$$ $\leftarrow$ comparing $\rightarrow$ find $A_\ell = 0$ $\quad B_\ell = 0$ when ℓ odd.

4

$P_1(\cos\theta) = 1$

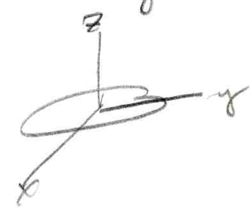
$$\phi(r,\theta) = \frac{q}{r} - \frac{a^2 q}{2}\frac{1}{r^3}P_2(\cos\theta) + \frac{3a^4 q}{8}\frac{1}{r^5}P_4(\cos\theta) - \frac{5a^6 q}{16}\frac{1}{r^7}P_6(\cos\theta) + \dots$$

$\Rightarrow B_0 = q/r \quad B_1 = 0 \quad B_2 = \frac{-a^2 q}{2}$

0 c) yes, because ϕ at $r=0$ will be most negative

4 d) $Q^{a} = \int_{C'} \lambda(\vec{r}')(3z'^2 - r'^2)\, d\vec{s}$

with origin at 0,0:



$$= \frac{q}{2\pi a}\int_0^{2\pi} r^2(3\cos^2\theta - 1)\, a\sin\theta\, d\phi$$

$r = a \quad \theta = 90°$

$$= \frac{q}{2\pi a}\int_0^{2\pi} -a^3\, d\phi = \boxed{-qa^2}$$

$3r^2\cos^2\theta - r^2 = r^2(3\cos^2\theta - 1)$

$r\sin\theta\, d\phi = a\sin 90°\, d\phi$

dr

$r\,d\theta$

$z = r\cos\theta$

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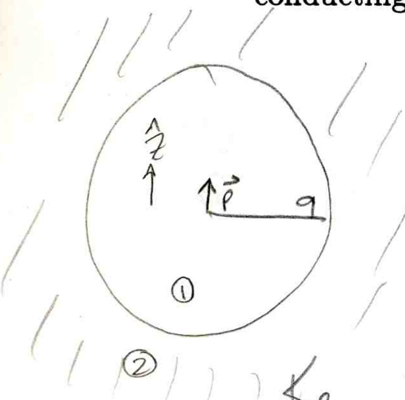
5. A point dipole $\vec{p}$ is place in the center of a spherical cavity of radius a in a dielectric with dielectric constant κ_e.

   a) Find the potential inside and outside the cavity.

   b) Use this result to find the potential for the case of a dipole inside a grounded conducting sphere of radius a.



I will call $\hat{z}$ the direction in which $\vec{p}$ is.

$$\phi(r,\theta) = \sum \left(A_\ell r^\ell + \frac{B_\ell}{r^{(\ell+1)}}\right) P_\ell(\cos\theta)$$ C

B.C.'s $r \to 0 \quad \phi \to \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \qquad r \to \infty \quad \phi \to 0$ C

at $r = a \quad D_{1n} = D_{2n}$ and ϕ is continuous

a) as $r \to 0$

$$\sum \left(\frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta) = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \qquad B_{\ell\neq 1} = 0 \qquad B_1 = \frac{p}{4\pi\epsilon_0}$$

r^{-2} $\quad -2r^{-3}$

$B_\ell \frac{\cos\theta}{r^2}$

$$\boxed{\begin{aligned} \phi_i &= \cancel{\sum} \frac{p\cos\theta}{4\pi\epsilon_0 r^2} + A_\ell r^\ell P_\ell \cos\theta \\ \phi_o &= \sum \frac{B_\ell}{r^{(\ell+1)}} P_\ell \cos\theta \quad \frac{1}{r^2} \end{aligned}}$$

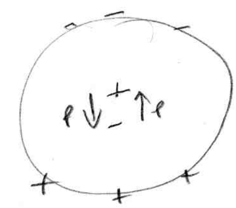
$$D_{ni} = -\epsilon_0\left(\frac{-2p\cos\theta}{4\pi\epsilon_0 r^3} + A_1\cos\theta\right)$$

$$D_{no} = -K_e\epsilon_0\left(\frac{p}{4\pi\epsilon_0} + A_1 a^3\right)\left(\frac{-2}{r^3}\right)\cos\theta$$

$$\phi_i(r=a) = \phi_o(r=a) \qquad \frac{p\cos\theta}{4\pi\epsilon_0 a^2} + \sum A_\ell a^\ell P_\ell(\cos\theta) = \sum \frac{B_\ell}{a^{\ell+1}} P_\ell(\cos\theta)$$

$\Rightarrow B_\ell = A_\ell = 0$ for $\ell \neq 1$ C

$$\frac{p\,\cancel{\cos\theta}}{4\pi\epsilon_0 a^2} + A_1 a\,\cancel{\cos\theta} = B_1 \frac{1}{a^2}\cancel{\cos\theta} \qquad B_1 = \frac{p}{4\pi\epsilon_0} + A_1 a^3$$



Work Space

$D_{n_i} = D_{n_o}$ at $r=a$

$$\cancel{\epsilon_0}\left(\frac{-2p\cancel{\cos\theta}}{4\pi\epsilon_0 r^3} + A_1 \cancel{\cos\theta}\right) = K_e \cancel{\epsilon_0}\left(\frac{p}{4\pi\epsilon_0} + A_1 a^3\right)\left(\frac{-2}{r^3}\right) \cancel{\cos\theta}$$

$$\frac{-2p}{4\pi\epsilon_0 r^3} + A_1 = \frac{-2pK_e}{4\pi\epsilon_0 r^3} - \frac{2K_e A_1 a^3}{r^3}$$

$$A_1 + \frac{2K_e a^3}{r^3}A_1 = \frac{2p}{4\pi\epsilon_0 r^3} - \frac{2pK_e}{4\pi\epsilon_0 r^3} \quad \text{at } r=a$$

$$A_1\left(1+\frac{2K_e a^3}{r^3}\right) = \frac{2p}{4\pi\epsilon_0 r^3}(1-K_e) \Rightarrow A_1 = \frac{2p}{4\pi\epsilon_0 a^3}\frac{(1-K_e)}{(1+2\cancel{\tfrac{a^3}{r^3}}K_e)}$$

$$A_1 = \frac{2p}{4\pi\epsilon_0 a^3}\frac{(1-K_e)}{(1+2K_e)}$$

$$\phi_i = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} + \frac{(1-K_e)}{(1+2K_e)}\frac{2p\cos\theta\, r}{a^3 4\pi\epsilon_0 \cancel{r^2}} \quad C$$

$$\phi_o = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} + \frac{(1-K_e)}{(1+2K_e)}\frac{2p\cos\theta}{4\pi\epsilon_0 r^2} = \frac{3}{1+2K_e}\frac{p}{4\pi\epsilon_0}\frac{\cos\theta}{r^2} \quad C$$

if surrounded by a conductor then $K_e = \infty$ and:

$$\phi_i = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} - \frac{1}{2}\left(\frac{2p\cos\theta}{a^3 4\pi\epsilon_0 r^2}\right) = \left(1-\frac{r^3}{a^3}\right)\frac{p\cos\theta}{4\pi\epsilon_0 r^2} \quad C \text{ almost}$$

$$\phi_o = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} - \frac{1}{2}\left(\frac{2p\cos\theta}{4\pi\epsilon_0 r^2}\right) = 0 \quad C$$

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$p = \int_S \sigma(\vec{r})\,\vec{r}\,d\vec{a}$

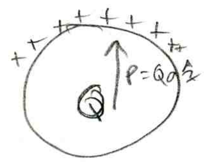
$p = Qa$

6. A conducting sphere of radius a, carrying a total charge Q, is placed in a previously uniform electric field $\vec{E} = [\frac{Q}{12\pi a^2 E_0}]\hat{z}$

   a) Find ϕ everywhere outside the sphere.

   b) Find the surface charge density σ everywhere on the sphere.

B.C. as $r \to \infty$



$\uparrow \vec{E} = \frac{Q}{12\pi a^2 \epsilon_0}\hat{z}$

$\phi \to -\frac{Q}{12\pi a^2 \epsilon_0} r\cos\theta$

at $r=a$ $\phi = 0$

$\phi = \sum \left[A_l r^l + B_l r^{-(l+1)}\right] P_l(\cos\theta)$

$0 = \sum \left[A_l a^l + B_l a^{-(l+1)}\right] P_l(\cos\theta) \Rightarrow B_l = -A_l a^{(2l+1)}$

$\phi(r,\theta) = \sum A_l \left(r^l - \frac{a^{(2l+1)}}{r^{(l+1)}}\right) P_l \cos\theta$

$r \to \infty$

$\sum A_l r^l P_l \cos\theta = -\frac{Q}{12\pi a^2 \epsilon_0} r\cos\theta + \frac{Q}{4\pi\epsilon_0 r}$

$\Rightarrow A_l = 0 \quad l \neq 1$ $\qquad B_l = 0 \quad l \neq 1$ $\qquad a^{2l+1-2} = a$

$A_1 = -\frac{Q}{12\pi a^2 \epsilon_0}$ $\qquad \Rightarrow B_1 = \frac{Q}{12\pi\epsilon_0} a^{(2l-1)}$ $\qquad E_r = -\frac{\partial}{\partial r}\phi$

$$\boxed{\phi(r,\theta) = -\frac{Q}{12\pi a^2 \epsilon_0} r\cos\theta + \frac{Qa}{12\pi\epsilon_0}\frac{1}{r^2}\cos\theta}$$

$\sigma = \epsilon_0 \vec{E}(\text{at } r=a)\cdot\hat{n} = \epsilon_0 E_r(r=a) \;//\; -\frac{\partial}{\partial r}\phi = \frac{Q}{12\pi a^2 \epsilon_2}$

SEE next page

Work Space

$$\phi = \sum \left(A_\ell r^\ell + \frac{B_\ell}{r^{(\ell+1)}}\right) P(\cos\theta)$$

$r \to \infty$

$$\phi \to -\frac{Q}{12\pi a^2 \epsilon_0} r\cos\theta + \frac{Q}{4\pi\epsilon_0 r}$$

$$A_\ell r^\ell P_\ell(\cos\theta) + \frac{B_\ell}{r^{-(\ell+1)}} P_\ell(\cos\theta) = -\frac{Q}{12\pi a^2\epsilon_0} r\cos\theta + \frac{Q}{4\pi\epsilon_0 r}$$

$$A_1 = -\frac{Q}{12\pi a^2\epsilon_0} \qquad B_0 = \frac{Q}{4\pi\epsilon_0}$$

$$B_1 = \frac{Qa}{12\pi\epsilon_0}$$

$$\boxed{\phi = -\frac{Qr\cos\theta}{12\pi a^2\epsilon_0} + \frac{Qa}{12\pi\epsilon_0 r^2}\cos\theta + \frac{Q}{4\pi\epsilon_0 r}}$$

C

$$E_r = -\frac{\partial}{\partial r}\phi = \frac{Q\cos\theta}{12\pi a^2\epsilon_0} \underset{+}{-} \frac{2Qa\cos\theta}{12\pi\epsilon_0 r^3} \underset{+}{-} \frac{Qa}{4\pi\epsilon_0 r^2}$$

$$\sigma_1 = \epsilon_0 E_r(r=a) = \frac{Q\cos\theta}{12\pi a^2} - \frac{Q\cos\theta}{6\pi a^2} - \frac{Q}{4\pi a^2}$$

close

$$= \frac{Q}{\pi a^2}\left(\frac{\cos\theta}{6} - \frac{1}{4}\right) = \boxed{\frac{Q}{6\pi a^2}\left(\cos\theta - \frac{3}{2}\right)}$$

$\frac{Q}{4\pi a^2}$

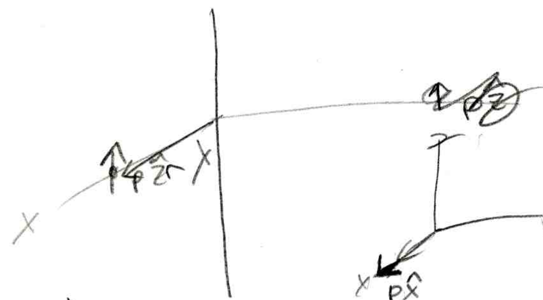
$$\frac{Q}{6\pi a^2}(\cos\theta + 1)$$

7. A point dipole of strength $\vec{p}$ lies a distance d away from an infinite line charge with linear charge density λ. For each of the three cases below, find the force $\vec{F}$ and torque $\vec{\tau}$ on the dipole. Assume that the line charge lies along the z-axis, and that the dipole sits at position $x = d,\ y = z = 0$. Be sure to give the direction and magnitude of $\vec{F}$ and $\vec{\tau}$.

a) Case 1: $\vec{p} = p\hat{z}$

b) Case 2: $\vec{p} = p\hat{x}$

c) Case 3: $\vec{p} = -p\hat{x}$



a) $\vec{F}_D = (\vec{p}\cdot\vec{\nabla})\vec{E}$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0\rho}\hat{\rho} = \frac{\lambda}{2\pi\epsilon_0\rho}(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})$$

since $\rho = x$, $\phi = 0$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{x}$$

$$\boxed{\vec{F}_D = p_x\frac{\partial}{\partial x}\left(\frac{\lambda}{2\pi\epsilon_0 x}\right)\hat{x} = 0}$$ C

because $\vec{p}\cdot\hat{x} = 0$ and $E_y\,E_z = 0$

$$\boxed{\vec{\tau} = \vec{p}\times\vec{E} = \frac{p\lambda}{2\pi\epsilon_0 d}\hat{y}}$$ C

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & p \\ \frac{\lambda}{2\pi\epsilon_0 x} & 0 & 0 \end{vmatrix} = \begin{matrix} \hat{x}\ 0 \\ \hat{y}\left(\frac{p\lambda}{2\pi\epsilon_0 x}\right) \\ \hat{z}\ 0 \end{matrix}$$

b) $\vec{F}_d = p_x\frac{\partial}{\partial x}\left(\frac{\lambda}{2\pi\epsilon_0 x}\right)\hat{x}$ $\boxed{p_x = p}$

$$= \boxed{-\frac{p\lambda}{2\pi\epsilon_0 d^2}\hat{x} = F_D}$$ C

$$\boxed{\vec{\tau} = \vec{p}\times\vec{E} = 0}$$ C

x^{-1} $-1x^{-2}$ x^3 $3x^2$

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c) $F_D = \frac{p\lambda}{2\pi\epsilon_0 d^2}\hat{x}$ C $p_x = -p$

$\vec{\tau} = 0$ C