Melinda Kellog

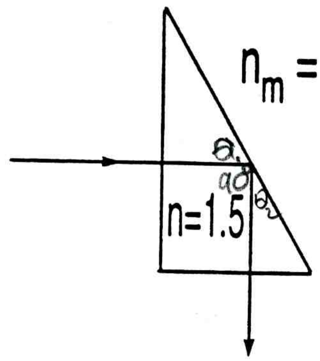
University of California

Physics 141 **Winter 1993**

MIDTERM EXAM

(Please show all of your work)

1. Right angle prisms are used to reflect light through 90° by means of total internal reflection as shown. Consider a prism made of glass with n=1.5 immersed in a medium of refractive index n_m. What is the maximum value n_m may have if the prism is to deflect all of the light by 90°?



$n_m = \frac{1.5}{\sqrt{2}}$ ✓

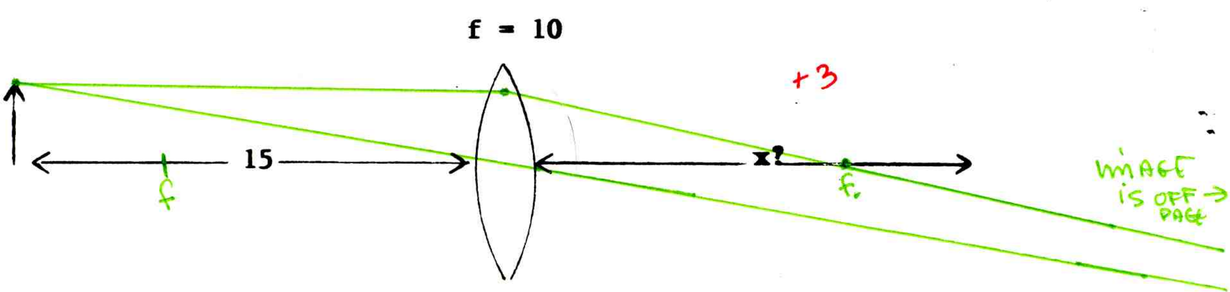
$\theta_1 = \theta_2 = 45°$

$\Rightarrow \theta_c = 45°$

10/10

2. a) Draw the ray tracing diagram for an object 15 cm before a lens of focal length f=10 cm.

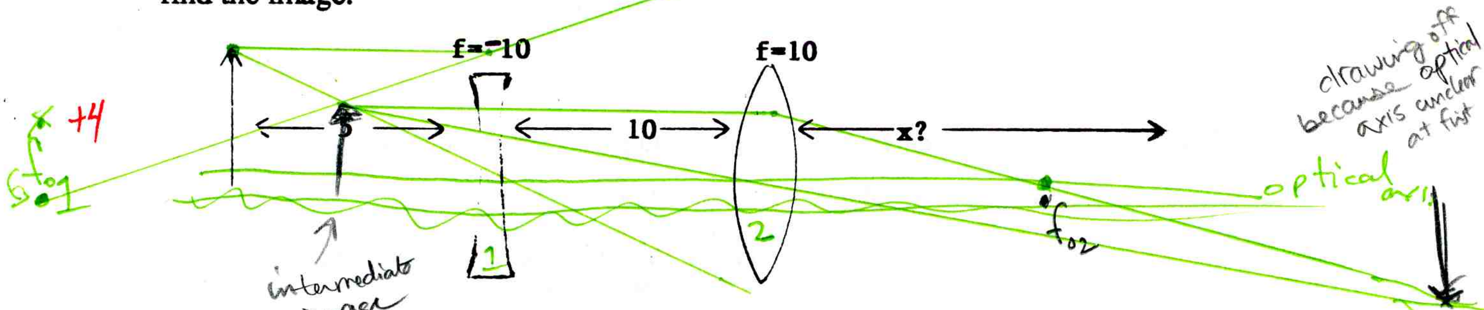
+3



+3 b) Determine the image location, x, and the magnification. 30 cm to the right of the lens. $M_T = -2$

c) A second lens, f=-10 cm, is added 10 cm before the first lens. Draw the ray diagram to find the image.

+4



image

+4 d) Verify your drawing by calculating the new image location and magnification.

New image at +40 cm from second lens

$M_T = -2$

+3 e) Is the image real or virtual?

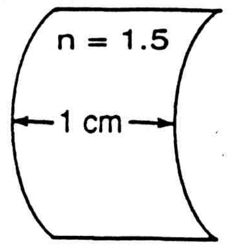
real

+3 f) How would one achieve the same image location and orientation with a single mirror?

Start with

3. A meniscus lens 1 cm thick ~~has~~ a refractive index n=1.5 and 10 cm radii of curvature as shown. Treat the lens as a thick lens.

R_1=10 cm R_2=10 cm

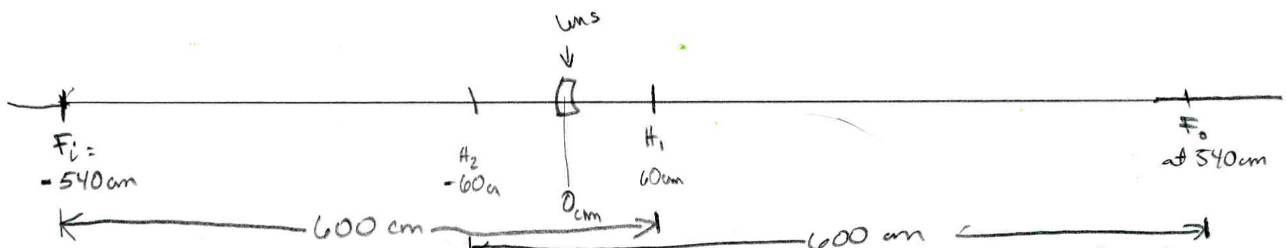


+5   a) What is the focal length of this lens? $f = 600$ cm

+4   b) Where are the principal planes located? $H_1 = +60$ cm $H_2 = -60$ cm

+5   c) Show the foci, the first and second principal points, and the lens in a sketch. Indicate distances clearly in cm. Label the object focus F_o and the image focus F_i.

14/15



4. An electric field polarized in the y-direction is a wave of amplitude 4 V/m, wavelength λ=500 nm, is traveling in the x-direction, and has value 2 V/m at the origin at time t=0:

$$|\mathbf{E}(x=0,y=0,z=0,t=0)| = 2$$

The wave is traveling in vacuum ($c=\omega/k=3\times10^8$ m/s)

a) Write this wave in the form $\mathbf{E}=\mathbf{E_o}\cos(\mathbf{k}\bullet\mathbf{r} - \omega t + \phi)$, where $\mathbf{r}=x\hat{x}+y\hat{y}+z\hat{z}$. What is E_o, k, ω, and φ?

+5 $\vec{E} = \hat{y}\, 4\left(\frac{V}{m}\right)\cos\left[\left(\frac{2\pi}{500}\times10^{9}\,m^{-1}\right)x - \left(\frac{6\pi}{500}\times10^{-1}\,s^{-1}\right)t + \frac{\pi}{3}\right]$

$E_o = 4\,V/m$
$K = \frac{2\pi}{500\,nm}$
$\omega = ck = \frac{2\pi}{500\,nm}\, 3\times10^8\,\frac{m}{s}$
$\phi = \frac{\pi}{3}$

b) What is the intensity of the light with this **E** field?

+4.5 $I \propto \langle E^2 \rangle = \frac{1}{2}E_0^2 = 8\,\frac{W}{m^2}$ $I = \frac{c\,\varepsilon_0}{2}|E_0|^2$

c) The wave passes into glass with n=1.4 . What are λ_g, ω_g, and k_g in the glass?

+5 $\lambda_g = \frac{500}{1.4}$ nm $\omega_g = \omega_0 = c\lambda_0 = \frac{2\pi}{500\,nm}\left(3\times10^8\frac{m}{s}\right)$ $k_g = \frac{2\pi}{\lambda_g} = \frac{2\pi(1.4)}{500\,nm}$

d) If $n(\lambda)=1.2+(5\times10^4\,nm^2)/\lambda^2$, what are the phase and group velocities in the glass?

phase velocity $= \frac{\omega_g}{k_g} = \frac{c\lambda_0\lambda_g}{2\pi} = \frac{(3\times10^8\,m/s)(500\,nm)/(\frac{500}{1.4}\,nm)}{2\pi}$

+5   group velocity $= \frac{\partial\omega}{\partial k} = v\left(1 - \frac{k}{n}\frac{\partial n}{\partial k}\right) = v\left[1 - \frac{k^2}{n}\left(\frac{2(5\times10^4\,nm^2)}{4\pi^2}\right)\right]$

where $k_g = \frac{2\pi(1.4)}{500\,nm}$ $n = 1.4$

20/20

5. The field due to a light bulb can be approximated as a spherical wave $E=(E_o/r)\exp(ikr-\omega t)$.

+4 **a) What is the intensity of light 1 m from a 100 W light bulb with λ=550 nm?** $\frac{100}{4\pi}$ W/m^2 ✓

+4 **b) What is the field amplitude at that point** $\frac{10}{\sqrt{\pi}}\frac{V}{m}$ ✓

+4 **c) What is the intensity at a point half way between two 100 W light bulbs 2 m apart?**

$I=I_1+I_2+2\sqrt{I_1I_2}\cos\delta$ Not counting interference since fluctuates so fast $I=\frac{200}{4\pi}$ W/m^2 ✓

+4 **d) What is the average electric field amplitude at that point?** $\sqrt{2}\,\frac{10}{\sqrt{\pi}}$ V/m ✓

e) What is the intensity at a point half way if instead of two light bulbs we have two in-phase lasers 2 m apart?

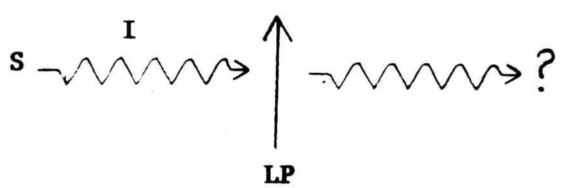
+4 $I=I_1+I_2+2\sqrt{I_1I_2}\cos\delta = 4I_0\cos^2\frac{\delta}{2}$ ✓

a $\lambda=550$ nm They will interfere constructively $\Rightarrow I=\frac{100}{\pi}$ W/m^2

6. An unpolarized laser beam with a coherence time $\Delta t_c \sim 10^{-3}$ s (and constant intensity I over a time scale < Δt_c) is to be used as the light source S below.

20/20

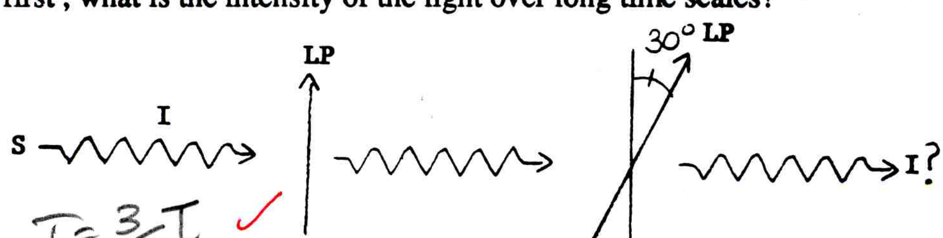
+3 **a) What is the nature of the intensity fluctuations observed when this light is passed through a linear polarizer (LP)? (Be brief)**



Every 10^{-3} seconds the intensity will change randomly (but remain constant that millisec) ✓

+3 **b) If the transmission axis of the LP is vertical, what is the intensity of the transmitted light averaged over long time scales (t >> Δt_c)?** $\frac{1}{2}I$ ✓

+3 **c) If a second LP is placed after the first LP with its transmission axis rotated 30° with respect to the first, what is the intensity of the light over long time scales?**



$I=\frac{3}{8}I$ ✓

d) For λ=500 nm, birefringent quartz has the following indices of refraction:

$n_o = 1.544$
$n_e = 1.553$

$1.553 - .544 = .009$

Calculate the thickness d of a quarter-wave plate made from this quartz. Is the optic-axis a fast or slow axis of the plate?

+2.5

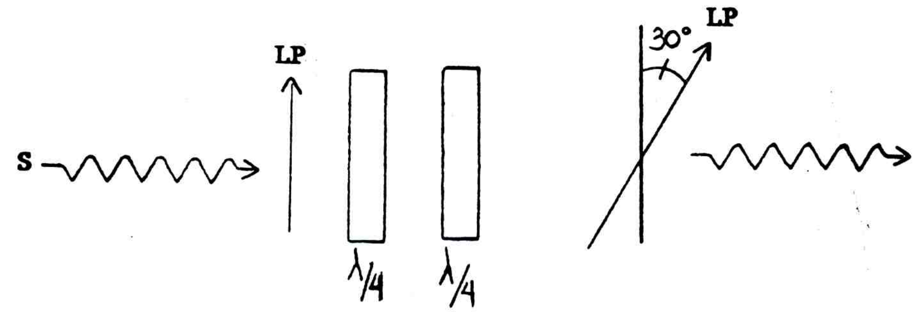
$\Delta\varphi=\frac{\pi}{2}$

$\Delta\varphi=\frac{2\pi}{\lambda}d(|n_e-n_o|)$

$d=\frac{\lambda}{4(.009)}=\boxed{\frac{500\text{ nm}}{.036}}$ ✓

~~fast axis~~ slow axis

e) If two of these quarter-wave plates are placed in series between the two LP's in part c), what is the polarization and intensity of the light transmitted through the optical system averaged over long time scales?



It depends on the orientation of the transmission axes of the $\frac{\lambda}{4}$ plates. ~~Let's say they were.~~ ~~If fast~~

Optic axis aligned with LP the

$$I = \frac{3}{8} I$$

See solutions. not exactly right.

+1.5

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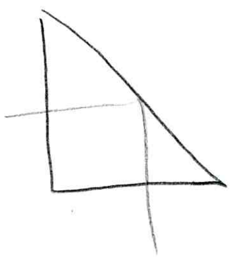
Melinda Kellog

1 $\sin\theta_c = \frac{n_t}{n_i}$

$n_i = 1.5$

$n_i \sin\theta_i = n_t \sin 90^\circ$

$\theta_c = 45^\circ$



$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad n_t = \frac{1.5}{\sqrt{2}}$

$\frac{10}{10}$

2. $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

$s_o = +15\text{ cm}$

$f = +10\text{ cm}$

$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{10\text{cm}} - \frac{1}{15\text{cm}}$

$s_i = \frac{10 \cdot 15\text{ cm}^2}{15 - 10\text{ cm}} = \frac{150}{5}\text{cm} = 30\text{ cm}$

$M_T = -\frac{s_i}{s_o} = -\frac{30}{15} = -2$

$$s_{i2} = \frac{df_2 - \frac{f_1 s_{o1} f_2}{s_{o1}-f_1}}{d - f_2 - \frac{s_{o1} f_1}{s_{o1}-f_1}}$$

$d = 10\text{ cm}$

$f_1 = -10\text{ cm}$

$f_2 = 10\text{ cm}$

$s_{o1} = 5\text{ cm}$

$\frac{100}{3}$ $\quad \frac{500}{15}\text{ cm}^2$

$$s_{i2} = \frac{10(10)\text{cm}^2 - \frac{-10(10)(5)\text{ cm}^3}{15\text{ cm}}}{10\text{cm} - 10\text{cm} - \frac{5\text{cm}(-10)\text{cm}}{15\text{ cm}}} = \frac{+100\text{ cm}^2 + \frac{100}{3}\text{ cm}^2}{+\frac{50}{15}\text{ cm}}$$

$\frac{10}{3}$

$$= \left(100 + \frac{100}{3}\right)\left(\frac{3}{10}\right)\text{ cm}$$

$$= 30 + 10 = \boxed{40\text{ cm}}$$

$$M_T = M_{T_1} M_{T2} = \left(+\cancel{\tfrac{10}{15}}\tfrac{2}{3}\right)(-3) = -2$$

$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$ $\quad f = -10$, $s_o = 5$

$$M_{T_1} = -\frac{s_i}{s_o} = +\frac{10}{3 \cdot 5} = +\frac{10}{15}$$

$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$

$$s_i = \frac{f s_o}{s_o - f} = \frac{-50}{15} = -\frac{10}{3}$$

$M_{T2} =$ $\quad s_o = \frac{10}{3} + 10$ $\quad M_{T2} = -\frac{s_i'}{s_o} = -\frac{40}{40/3} = -3$

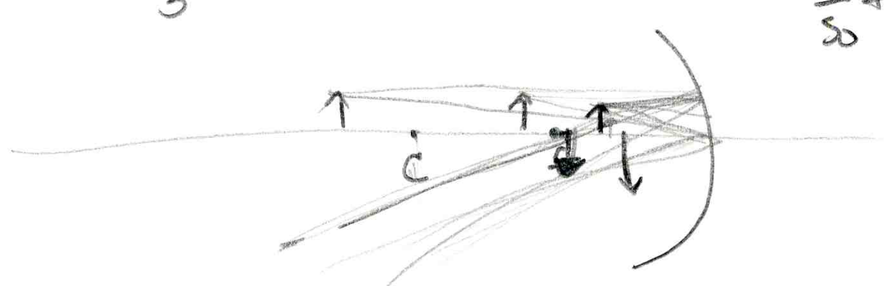
$s_i = \frac{f s_o}{s_o - f}$ $\quad f = +10$

$10 \quad \frac{30}{3}$

$$s_i = \frac{10\left(10 + \frac{10}{3}\right)}{\frac{10}{3}} = 3\left(10 + \frac{10}{3}\right) = 30 + 10 = 40\text{ cm}$$

$s_o = \frac{1}{2} f$

$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$



$$s_i = \frac{f \frac{1}{2} f}{-\frac{1}{2}} = -f^2$$

$\frac{1}{s_i} = (n_e - 1)$

$$\frac{1}{s_o} + \frac{1}{s_i} = \left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_e - 1)d}{n_e R_1 R_2}\right)(n_e - 1)$$ ✓

$n_e = 1.5$

$R_1 = +10\,cm$ $\qquad$ $d = 1cm$

$R_2 = +10\,cm$

$$\frac{1}{f} = (n_e - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_e - 1)d}{n_e R_1 R_2}\right] = \frac{1}{2}\left[\cancel{\frac{1}{10} - \frac{1}{10}} + \frac{\frac{1}{2}\,1cm}{\frac{3}{2}\,100cm}\right] = \frac{1}{2}\frac{1}{3}\frac{1}{100cm}$$

$f = 600\ cm$ $\qquad$ +5

$\cancel{h_1 = \frac{f_1(d - f_2)}{d - (f_1 + f_2)} =}$

$$\overline{OH_1} = \frac{fd}{R_2} = \frac{600cm\ 1cm}{10cm} = 60cm$$

$$\overline{OH_2} = -\frac{fd}{R_1} = \frac{-600cm\ 1cm}{10cm} = -60\,cm$$

$$h_1 = -\frac{f(n-1)d}{R_2 n}$$ $\qquad$ Close

$$h_2 = -\frac{f(n-1)d}{R_1 n}$$

$_{0y} = 4 \frac{V}{m}$ $\lambda = 500\,nm$ $k = \frac{2\pi}{\lambda}$

$\omega = ck$

$\varphi_0 \Rightarrow \cos(kx - \omega t + \varepsilon) = \frac{1}{2}$

at $t=0$ $x=0$

$$\vec{E} = \hat{y}\, 4 \frac{V}{m} \cos\left(\frac{2\pi}{500nm} x - \left(8\times10^{3} \frac{m}{s}\right)\left(\frac{2\pi}{500nm}\right) t + \frac{\pi}{3}\right)$$

$\cos \varepsilon = \frac{1}{2}$

$\varepsilon = +\frac{\pi}{3}$

$kx = \frac{\pi}{3}$

$$\frac{2\pi\, \cancel{8}^{3} \times \cancel{10^{8}}\, 10^{-1}}{500 \times \cancel{10^{-9}}} \quad s^{-1}$$

c) $\lambda_0 = 500\,nm$

$$\lambda_g = \frac{500\,nm}{1.4} =$$

$$n(\lambda) = 1.2 + \frac{5\times10^4\,nm^2}{\lambda^2} \Rightarrow n(k) = 1.2 + \frac{5\times10^4\,nm^2\; k^2}{4\pi^2}$$

$$\frac{dn}{\partial k} = \frac{2(5\times10^4\,nm^2)}{4\pi^2} k \qquad \lambda = \frac{2\pi}{k}$$

$k = 2\pi/\lambda$

$$\frac{\partial \omega}{\partial k} = v\left[1 - \frac{k^2}{n}\left(\frac{2(5\times10^4\,nm^2)}{4\pi^2}\right)\right]$$

a) Intensity 1 m from 100W w/ $\lambda = 550\,nm$

Intensity $= \frac{P}{A}$ at 1m $A = 4\pi r^2$
$= 4\pi$

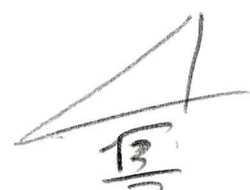
$\Rightarrow I = \frac{100}{4\pi} \, W/m^2$

b) Field amplitude is $2\sqrt{I} =$

$I = \langle E^2 \rangle = \frac{1}{2} E_0^2$ $\frac{10}{2\sqrt{\pi}} = \frac{5}{\sqrt{\pi}} = \frac{10}{\sqrt{\pi}}$ ✓

c)

e) $\frac{1 \text{ meter}}{550 \times 10^{-9} m} = \frac{1}{550} \times 10^{9}$ wavelengths assumes $\lambda = 550\,nm$ is exact



6 c) $I = I_0 \cos^2 30$
$= \frac{I}{2} \cos^2 30^\circ = \frac{I}{2} \cdot \frac{3}{4} = \frac{3}{8} I$

1	10
2	20
3	14
4	20
5	20
6	13
	97/100

– If you have time, would you please see me after class?

Thanks.

– [illegible]